Rule 1. Same Base

$$a^m a^n = a^{m+n}$$

"To multiply powers of the same base, add the exponents."

For example, $a^2a^3 = a^5$

$$x^3 \cdot x^{-2} = x^1$$

Rule 2: Subtracting exponents

$$\frac{a^m}{a^n} = \begin{cases} a^{m-n} & \text{if } m > n. \\ \frac{1}{a^{n-m}} & \text{if } n > m. \end{cases}$$

$\frac{x^5}{x^2}$

x^{5-2}

x^3

$\frac{x^2}{x^7}$

$= \frac{1}{x^{7-2}}$

$\frac{1}{x^5}$

Rule 3: Power of a Power

$$(a^m)^n = a^{mn}$$

"To take a power of a power, multiply the exponents."

$$(x^5)^3 = x^{15}$$

$$x^5 \cdot x^5 \cdot x^5$$

Rule 4: Power of a Product of Factors

$$(ab)^n = a^n b^n$$

"Raise <u>each</u> factor to that same power."

$$(2x^3)^3$$

$$2^3 x^9 = 8x^9$$

Rule 5: Power of a fraction

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad \rightarrow \quad \left(\frac{x^3}{y^2}\right)^3 = \frac{x^9}{y^6}$$

"To raise a fraction to a power, raise the numerator and denominator to that power."

Special note $b \neq 0$

Rule 6: a number raised to a *negative* exponent is the same as the reciprocal of that power with a positive exponent.

$a^{-n} = \frac{1}{a^n}$

$(13x)^{-1}$

$13x^{-1} \Rightarrow 13 \cdot \frac{1}{x}$

$\frac{1}{13x}$

a^{-n} is the *reciprocal* of a^n.

$\frac{13}{x}$

$$a \cdot a^{-1} = 1$$

Rule 7: Exponent 0

Any number (except 0) with exponent 0 is defined as 1.

$$a^0 = 1$$

$\frac{5^3}{5^3} = 5^{3-3} = 5^0$

$\frac{0}{0}$ undefined

$0^0 = 1$

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now we will mix them up

#1 $(x^{-2}y)^2\left(\frac{2}{x}\right)^{-2}$

$\left(\frac{y}{x^2}\right)^2\left(\frac{x}{2}\right)^2$

$\frac{y^2}{x^4}\cdot\frac{x^2}{4} = \frac{y^2}{4x^2}$

x^2

$$3^{-1}\left(\frac{4^{-2}}{3-3^{-2}}\right)$$

$$\frac{1}{3}\left(\frac{\frac{1}{4^2}}{3-\frac{1}{3^2}}\right) = \frac{1}{3}\left(\frac{\frac{1}{16}}{\frac{3}{1}-\frac{1}{9}}\right)$$

$$\frac{1}{3}\left(\frac{\frac{1}{16}}{\frac{27}{9}-\frac{1}{9}}\right)$$

$$\frac{1}{3}\left(\frac{\frac{1}{16}}{\frac{26}{9}}\right)$$

$$\frac{1}{3}\cdot\frac{1}{16}\cdot\frac{9}{26}$$

$$\frac{3}{416}$$

5) $x^7 x^{-4} = x^3$

7) $2a^2a^{-6} = \frac{2a^2}{a^6}$

$= \frac{2}{a^4}$

9 $5 \times 5^{-3} = \frac{1}{25}$

$\frac{5}{5^3}$ $\frac{1}{25}$

11) $(2\pi x^{-1})^2$

$$\left(\frac{2\pi}{x}\right)^2 = \frac{4\pi^2}{x^2}$$

$$(5an^{-2})^{-1}$$

$$\left(\frac{5a}{n^2}\right)^{-1}$$

$$\frac{n^2}{5a}$$

29) $3x^{-2} + 2y^{-2}$

y^2 $\frac{3}{x^2} + \frac{2}{y^2}$ x^2 x^2

$$\frac{3y^2 + 2x^2}{x^2y^2}$$

31) $(2a^{-n})^2 \left(\frac{3}{2a^n}\right)^{-1}$

$\left(\frac{2}{a^n}\right)^2 \cdot \frac{2a^n}{3}$

$\frac{4}{a^{2n}} \cdot \frac{2a^n}{3} = \frac{8a^n}{3a^{2n}}$

$\frac{8}{3a^{2n-1n}}$

$\frac{8}{3a^n}$

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$$2a^{-2} + (2a^{-2})^4$$

$$\frac{2}{a^2} + \left(\frac{2}{a^2}\right)^4$$

a^6 $\quad \frac{2}{a^2} + \frac{16}{a^8}$ $\qquad \frac{2a^6 + 16}{a^8}$

a^6

45 $\dfrac{6^{-1}}{4^{-2}+2} = \dfrac{\frac{1}{6}}{\frac{1}{4^2}+2}$

$\dfrac{\frac{1}{6}}{\frac{1}{16}+2} = \dfrac{\frac{1}{6}}{\frac{1}{16}+\frac{32}{16}}$

$\dfrac{\frac{1}{6}}{\frac{33}{16}} = \dfrac{1}{\cancel{6}_{3}} \cdot \dfrac{\cancel{16}^{8}}{33}$

$\dfrac{8}{99}$

$$\frac{x^{-2}-y^{-2}}{x^{-1}-y^{-1}} = \frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}-\frac{1}{y}}$$

$$\frac{\frac{y^2-x^2}{x^2y^2}}{\frac{y-x}{xy}} \qquad 1 \qquad \frac{(y-x)(y+x)}{x^2y^2} \cdot \frac{xy}{y-x}$$

$$\frac{y+x}{xy}$$