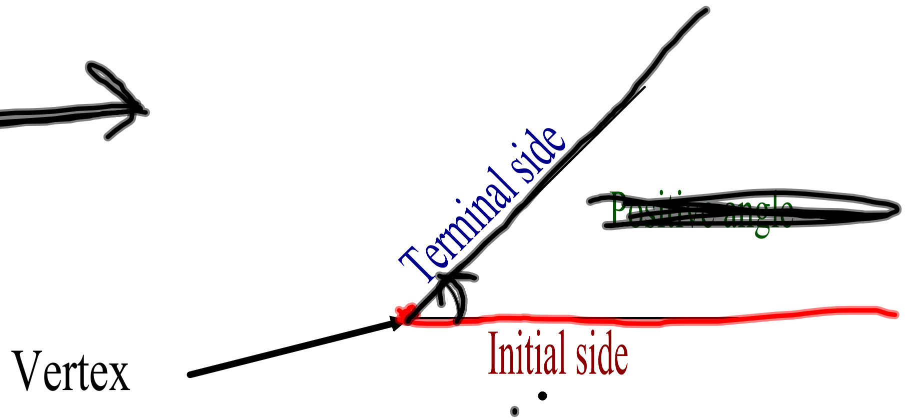
4.1 Angles

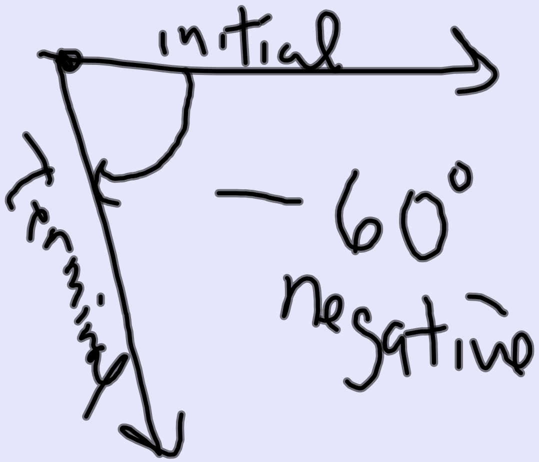
An angle is formed rotating a ray about a fixed endpoint from the initial position to a terminal position .

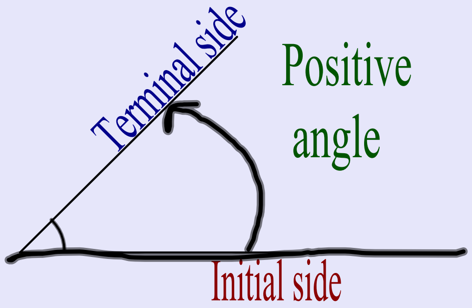


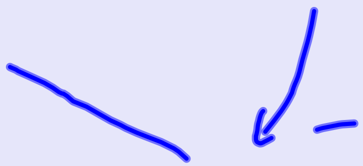
Negative angle-
clockwise rotation from the
initial side to the terminal side.

Positive angle-
counterclockwise rotation from the
initial side to the terminal side.

55°



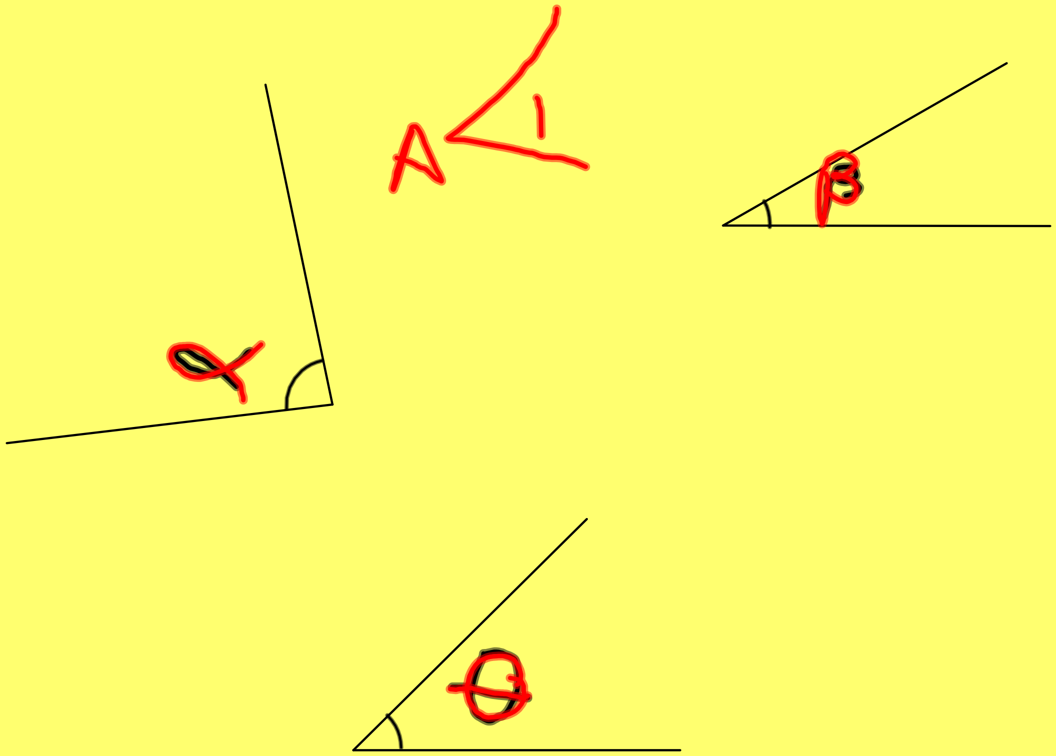




Different symbols to designate angles

α(alpha), β(beta), ϕ(Phi), θ(theta)

fi



Decimal Degree and Degree-Minute-Second Conversion

$\angle s$ $1^\circ = \frac{1}{360}$ of a complete rotation.

Degree measure of an angle can be expressed more precisely by using decimal degrees, or degrees, minutes, and seconds.

1 degree (1°) = 60 minutes ($60'$) = 3600 seconds ($3600''$)
1 minute ($1'$) = 60 seconds ($60''$)

convert from degrees to DMS

47.5° $47^\circ 30'$

47° $.5 \times 60 = 30'$

52.35°

$52^\circ 21'$ $.35 \times 60 = 21'$

convert from DMS to degrees

$35^\circ 15' 20''$

$35^\circ + \frac{15}{60} + \frac{20}{3600}$

$35^\circ + .25 + .006$

35.256°

3600

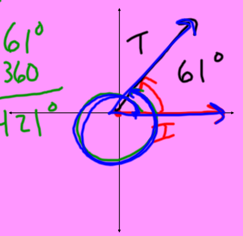
$23^\circ + .41 + .009$

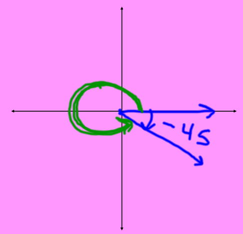
23.429

Coterminal angles

Def: Angles that have the same initial and terminal sides.

$61^\circ + 360 = 421^\circ$





Finding negative and positive coterminal angles.

$50^\circ + 360^\circ = 410^\circ$

$410^\circ + 360^\circ = 770^\circ$

$50^\circ - 360^\circ = -310^\circ$

$-310^\circ - 360^\circ = -670^\circ$

30°

18° 732°

91° 451°

Are $\theta_1 = 91^\circ$ and $\theta_2 = 451^\circ$ coterminal?

$91^\circ + 360^\circ = 451^\circ$

Yes.

Determine a positive coterminal angle with a -540° angle.

$-540^\circ + 360^\circ = -180^\circ$

$-180^\circ + 360^\circ = 180^\circ$

changing radians to degrees and back using the definition or the calculator.

π^r

Def: $\boxed{\pi \text{ rad} = 180^\circ}$

Change radians to degrees

$\frac{\pi^r}{\pi} = \frac{180^\circ}{\pi}$

$1^r = \frac{180^\circ}{\pi}$

$\boxed{n^r = \left(n \cdot \frac{180}{\pi}\right)^\circ}$

Change degrees to radians

$\frac{\pi}{180}^r = \frac{180^\circ}{180}$

$\frac{\pi}{180}^r = 1^\circ$

$\boxed{n^\circ = \left(n \cdot \frac{\pi}{180}\right)^r}$

$1.57^r = 1.57 * \frac{180}{\pi}$

$1.57^r = 89.95$

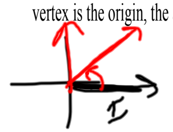
$180 \times \frac{\pi}{180}^r$

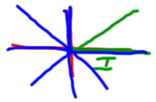
$65^\circ \times$

$3.14192^r = 3.14192 \times \frac{180}{\pi}$

Standard position of an angle

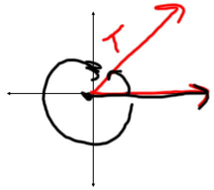
If the initial side of the angle is the positive x-axis and the vertex is the origin, the angle is in **standard position**



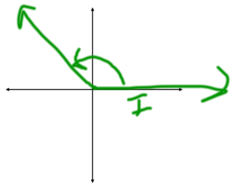


$\left(\frac{\pi}{2}\right)^r$

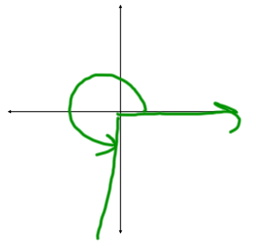
If the terminal side is in the first quadrant, the angle is called a **first quadrant angle.**



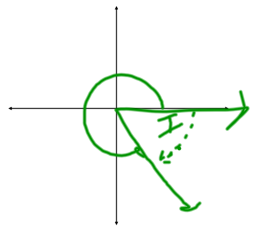
If the terminal side is in the second quadrant, the angle is called a **second quadrant angle.**

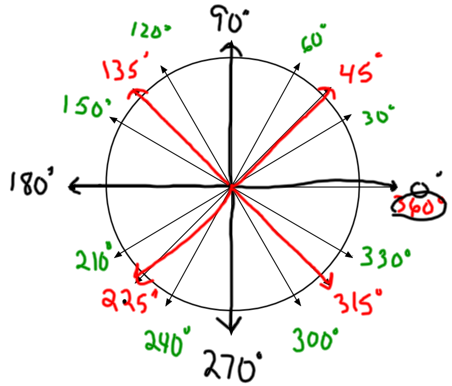


If the terminal side is in the third quadrant, the angle is called a **third quadrant angle.**

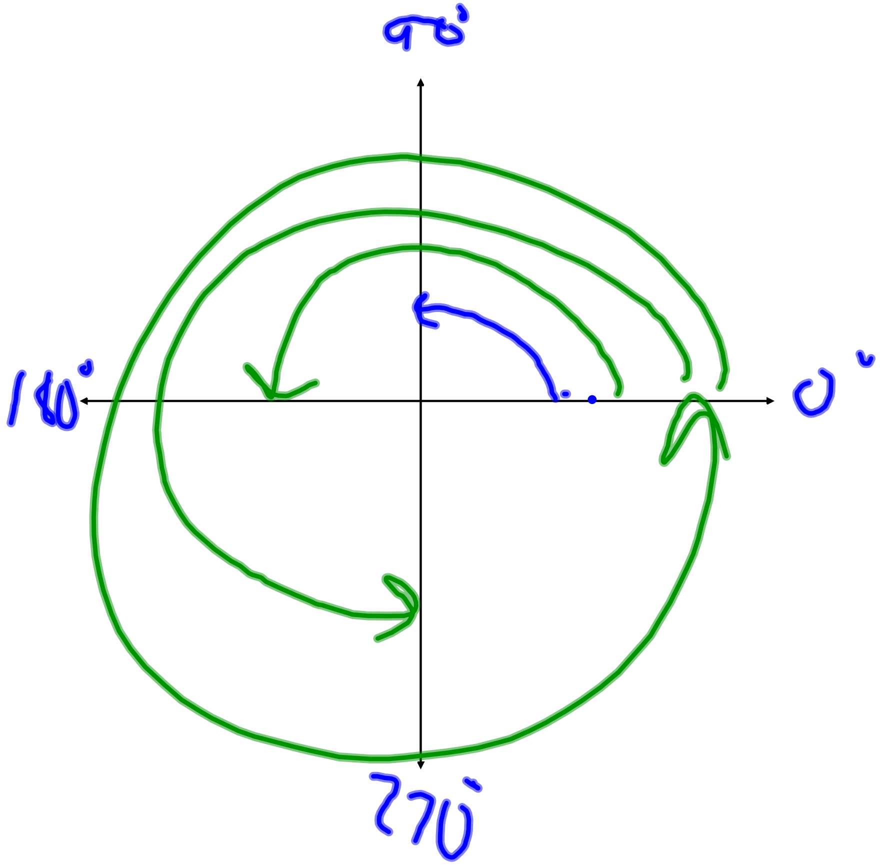


If the terminal side is in the fourth quadrant, the angle is called a **fourth quadrant angle.**



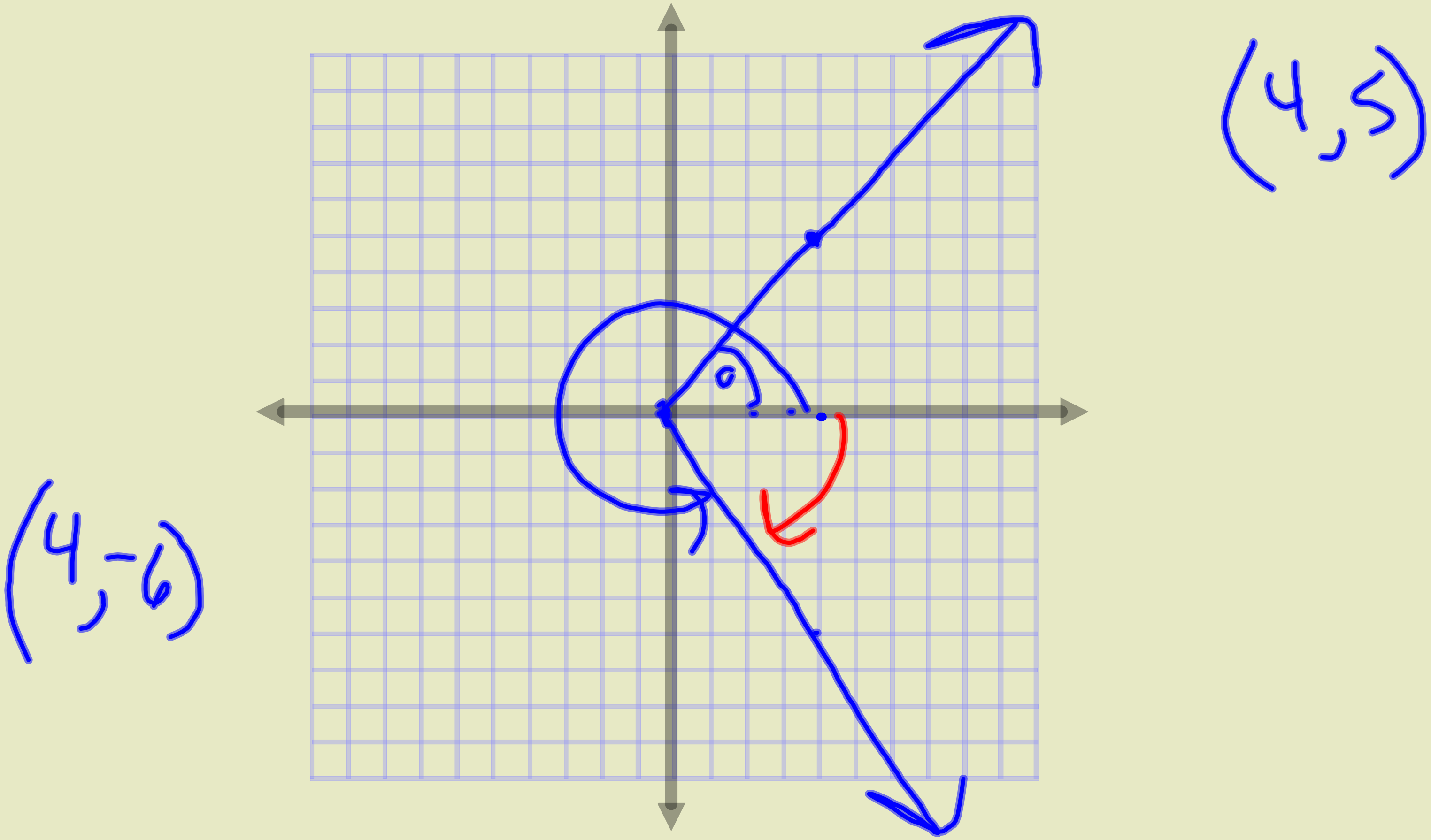


If the terminal side coincides with one of the axis, the angle is called a **quadrantal angle.**

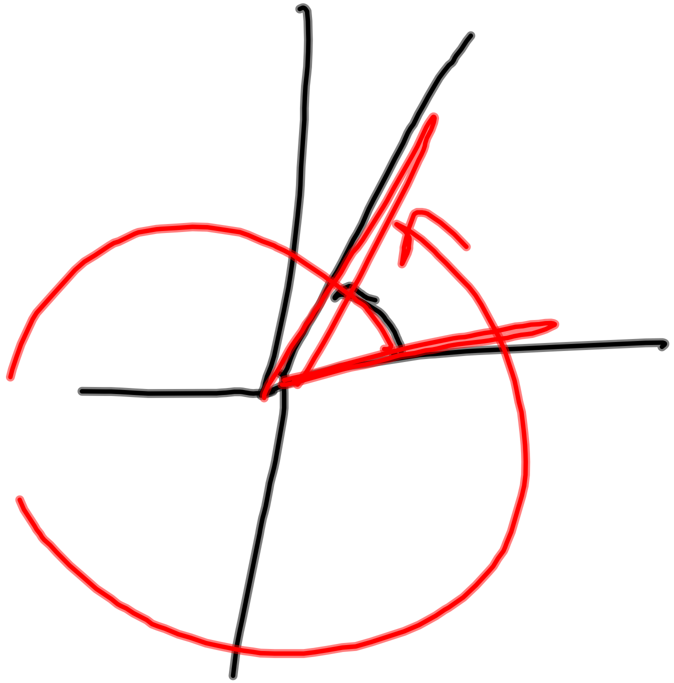


If the terminal side passes through a point.

(we can determine the terminal side of an angle)



$$\begin{array}{r} 70^\circ\ 30' \\ +360 \\ \hline 430^\circ\ 30' \end{array}$$



$$\begin{array}{r} {}^{3}\ {}^{60} \\ \cancel{4}:00 \\ -2\ 20 \end{array}$$

$$\begin{array}{r} {}^{59}\quad 60' \\ -36\cancel{0} \\ 70^\circ\ 30' \\ \hline -289^\circ\ 30' \end{array}$$

$$\begin{array}{r} 310 \\ -\ 38 \\ \hline \end{array}$$

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