5.5 Solving Systems of Two linear Equations in Two Unknowns by Determinants

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

standard form

Another method to solve 2 equations in 2 unknowns is by using determinants:
If a, b,and c, are any three real numbers, then the symbol

$$\begin{array}{ll} \text{row} \rightarrow & \\ \text{row} \rightarrow & \end{array} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

columns

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$ is called a 2 by 2 matrix

$[A]_{2 \times 2}$ row column

A

A determinant is a square array of numbers or variables (elements).
The vertical bars that close the array show its a determinant .
below is a second order determinant

the number $a_1b_2 - a_2b_1$ is called the value of the determinant

principal diagonal

secondary diagonal

$$\det \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

evaluate the given determinants

$$\det \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = 2-12 = -10$$

$$\det \begin{vmatrix} -5 & 1 \\ 0 & 3 \end{vmatrix} = -15-0 = -15$$

$$\det \begin{vmatrix} -6 & 3 \\ -4 & 2 \end{vmatrix} -12 + +12 = 0$$

$$\det \begin{vmatrix} 4 & 2 \\ 6 & 8 \end{vmatrix} = 32-12 = 20$$

We can use Cramer's rule to solve two linear equations in 2 unknowns.

1. the equations have to be in standard form.
2. the determinant of the denominators are the coefficients of x and y
3. the determinant of the numerator to find x, replace the x coefficients with the constants.
4. the determinant of the numerator to find y, replace the y coefficients with the constants.

the solution to the system

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}$$

is (x,y)

determinant of the coefficients is the denominator

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

determinant of the numerator for x

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

determinant of the numerator for y

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Solve the given systems of equation by determinants

$$1x + 3y = 7$$
$$2x + 3y = 5$$

$$D = \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 3 - 6 = -3$$

$$D_x = \begin{vmatrix} 7 & 3 \\ 5 & 3 \end{vmatrix} = 21 - 15 = 6$$

$$D_y = \begin{vmatrix} 1 & 7 \\ 2 & 5 \end{vmatrix} = 5 - 14 = -9$$

$$x = \frac{D_x}{D} = \frac{6}{-3} = -2$$

$$y = \frac{D_y}{D} = \frac{-9}{-3}$$

$$(-2, 3)$$

Solve the given systems of equation by determinants

$$1x - 4y = 17$$
$$3x + 4y = 3$$

$$D = \begin{vmatrix} 1 & -4 \\ 3 & 4 \end{vmatrix} \quad 4+12=16$$

$$Dx = \begin{vmatrix} 17 & -4 \\ 3 & 4 \end{vmatrix} \quad 68+12=80$$

$$Dy = \begin{vmatrix} 1 & 17 \\ 3 & 3 \end{vmatrix} \quad 3-51=-48$$

$$x = \frac{Dx}{D} = \frac{80}{16} = 5$$

$$y = \frac{Dy}{D} = \frac{-48}{16} = -3$$

POI $(5,-3)$

Solve the given systems of equation by determinants

$3i_1 + 5 = -4i_2$

$3i_2 = 5i_1 - 2$

$\rightarrow 3i_1 + 4i_2 = -5$
$-5i_1 + 3i_2 = -2$

$D = \begin{vmatrix} 3 & 4 \\ -5 & 3 \end{vmatrix} = 9 + {}^{+}20$
29

$D_x = \begin{vmatrix} -5 & 4 \\ -2 & 3 \end{vmatrix} = -15 + {}^{+}8 = -7$

$D_y = \begin{vmatrix} 3 & -5 \\ -5 & -2 \end{vmatrix}$
$= -6 + {}^{-}25$
-31

$x = \frac{D_x}{D}$ $\quad y = \frac{D_y}{D}$

$x = \frac{-7}{29}$ $\quad y = \frac{-31}{29}$

Solve the given systems of equations by determinants

a_1 b_1 c_1

$2.5\,x + 2.25\,y = 4.00$

$3.75\,x - 6.75\,y = 3.25$

a_2 b_2 c_2

$D = \begin{vmatrix} 2.5 & 2.25 \\ 3.75 & -6.75 \end{vmatrix}$

$2.5 \times -6.75 = -16.9$

$3.75 \times 2.25 = 8.4$

$-16.9 - 8.4 = \underline{-25.3}$

$D_x = \begin{vmatrix} 4.00 & 2.25 \\ 3.25 & -6.75 \end{vmatrix}$

$4.00 \times -6.75 = -27$

$2.25 \times 3.25 = 7.31$

$-27 - 7.31 = \underline{-34.31}$

$D_y = \begin{vmatrix} 2.5 & 4.00 \\ 3.75 & 3.25 \end{vmatrix}$

$2.5 \times 3.25 = 8.125$

$4.00 \times 3.75 = 15$

$8.125 - 15 = \underline{-6.9}$

$\dfrac{-34.31}{-25.3} = 1.356$ $\quad x = 1.356$

$\dfrac{-6.9}{-25.3} = \quad y = .\overline{27}$

***Solve the given system of equations by determinants

$2x + 6y = -3$

$-6x = 18y + 5$

$-18y \quad -18y$

$-6x - 18y = 5$

$$D = \begin{vmatrix} 2 & 6 \\ -6 & -18 \end{vmatrix} -36 + 36 = 0$$

$$Dx = \begin{vmatrix} -3 & 6 \\ 5 & -18 \end{vmatrix} 54 - 30 = 24$$

$$Dy = \begin{vmatrix} 2 & -3 \\ -6 & 5 \end{vmatrix} 10 - 18 = -8$$

$$x = \frac{24}{0}$$

$$y = \frac{-8}{0}$$

Parallel!

***Solve the given system of equations by determinants

$3x - y = 5$

$-9x = -3y - 15$

$+3y \quad +3y$

$-9x + 3y = -15$

$x = \frac{0}{0}$

$y = \frac{0}{0}$

$D = \begin{vmatrix} 3 & -1 \\ -9 & 3 \end{vmatrix} \; 9 - 9 = 0$

$Dx = \begin{vmatrix} 5 & -1 \\ -15 & 3 \end{vmatrix} \; 15 - 15 = 0$

$Dy = \begin{vmatrix} 3 & 5 \\ -9 & -15 \end{vmatrix} \; -45 + 45 = 0$

Same line

infinite Solutions

$\frac{Dy}{D} = \frac{-}{0}$