5.6 Solving Systems of Three Linear Equations in Three Unknowns Algebraically

Standard form:
$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

idea: reduce the system to a 2 by 2 system.

the strategy is to reduce this to two equations in two unknowns

1. Eliminating one of the unknowns from two pairs of equations
2. eliminate the same unknown from another pair of equations
3. use the elimination, substitution or Cramer's rule method to solve for the two variables.
4. substitute the value for the two variables to find the third
5. check your answers

solve this system of three equations in three unknowns

$$\begin{cases} x + y - z = 4 & (1) \\ x - 2y + 3z = -6 & (2) \\ 2x + 3y + z = 7 & (3) \end{cases}$$

2nd Method : By Elimination

→ Form two different pairs of equations and eliminate the same unknown in both pairs.

$$\begin{array}{l} (1) \\ (2) \end{array} \begin{cases} 3 \cdot x + 3 \cdot y - 3 \cdot z = 3 \cdot 4 \\ x - 2y + 3z = -6 \end{cases}$$

$$\begin{cases} 3x + 3y - 3\cancel{z} = 12 \\ x - 2y + 3\cancel{z} = -6 \end{cases}$$

$$4x + y = 6$$

$$\begin{array}{l} (1) \\ (3) \end{array} \begin{cases} x + y - \cancel{z} = 4 \\ 2x + 3y + \cancel{z} = 7 \end{cases} \text{ Add}$$

$$3x + 4y = 11$$

$$\begin{cases} 3x + 4y = 11 \\ 4x + y = 6 \end{cases}$$

solve this system of three equations in three unknowns

$$x + y - z = -3$$
$$x + z = 2$$
$$2x - y + 2z = 3$$

$x + y - z = ^{-}3$

$2x - y + 2z = 3$

$3x + z = 0$

$-\ x + z = 2$

$2x = ^{-}2$

$\frac{2x}{2} = \frac{^{-}2}{2}$

$x = ^{-}1$

find z

$x + z = 2$

$-1 + z = 2$

$z = 3$

find y

$x + y - z = ^{-}3$

$-1 + y - 3 = -3$

$y - 4 = -3$

$y = 1$

$$\begin{cases} x = ^{-}1 \\ y = 1 \\ z = 3 \end{cases}$$

show that the given systems of equations have either an unlimited number of solutions or no solution. If there is an unlimited number of solutions, find one of them

1st $x - 2y - 3z = 2$

2nd $x - 4y - 13z = 14$

3rd $-3x + 5y + 4z = 2$

Combined 1st + 2nd equations $-2y - 10z = 12$

$3x - 6y - 9z = 6$ 1st

$-3x + 5y + 4z = 2$ 3rd

$-y - 5z = 8$

$-2y - 10z = 12 \rightarrow -2y - 10z = 12$

Times -2 $\rightarrow$ $-y - 5z = 8 \rightarrow 2y + 10z = 16$

$0 = 28$

Parallel lines

show that the given systems of equations have either an unlimited number of solutions or no solution. If there is an unlimited number of solutions, find one of them

$3x + y - z = -3$
$x + y - 3z = -5$
$-5x - 2y + 3z = -7$

$-3x + \cancel{y} - z = -3$
$x + \cancel{y} - 3z = -5$
$-2x - 2z = -2$

Times 2
$x + y - 3z = -5$
$-5x - 2y + 3z = -7$

$2x + 2y - 6z = -10$
$-5x - 2y + 3z = -7$
$-3x - 3z = -17$

times 2 $-3x - 3z = -17$
times 3 $-2x - 2z = -2$

$-6x - 6z = -34$
$6x + 6z = 6$

no solution

$0 = -28$

period 11 only

solve this system of equations

$$3x + 2y - 4z + 2t = 3$$

$$5x - 3y - 5z + 6t = 8$$

$$2x - y + 3z - 2t = 1$$

$$-2x + 3y + 2z - 3t = -2$$

$) \cdot -3$

$$-9x - 6y + 12z - \cancel{6t} = -9$$

$$5x - 3y - 5z + \cancel{6t} = 8$$

$$-4x - 9y + 7z = -1$$